

Summer + Math = (Best Summer Ever)²

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

$$f(-2) = (-2)^2 - 2(-2) + 5$$

$$f(-2) = 4 + 4 + 5$$

$$f(-2) = 13$$

2. $f(x+2) =$

$$x^2 + 2x + 5$$

3. $f(x+h) =$

$$(x+h)^2 - 2(x+h) + 5$$

$$(x+h)(x+h) - 2x - 2h + 5$$

$$x^2 + xh + xh + h^2 - 2x - 2h + 5$$

$$x^2 + 2xh + h^2 - 2x - 2h + 5$$

Use the graph $f(x)$ to answer the following.

4. $f(0) = -4$

$$f(4) = \text{DNE (Does not exist or undefined)}$$

$$f(-1) = -3.5$$

$$f(-2) = -2$$

$$f(2) = \text{DNE (Does not exist or undefined)}$$

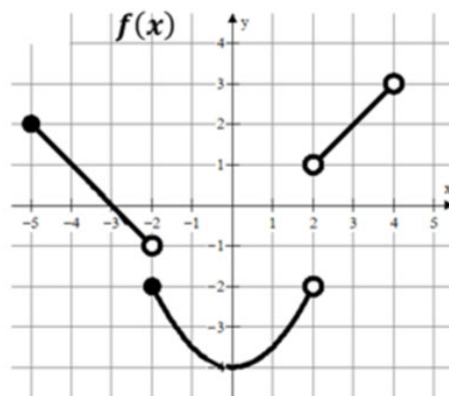
$$f(3) = 2$$

$$f(x) = 2 \text{ when } x = ?$$

-5 and 3

$$f(x) = -3 \text{ when } x = ?$$

-1.5 and 1.5



Write the equation of the line meets the following conditions. Use point-slope form.

$$y - y_1 = m(x - x_1)$$

5. slope = 3 and (4, -2)

$$y - (-2) = 3(x - 4)$$

$$y + 2 = 3(x - 4)$$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

$$y - 7 = -\frac{3}{2}(x + 5)$$

7. $f(4) = -8$ and $f(-3) = 12$

$$m = \frac{12 - (-8)}{-3 - 4} = \frac{20}{-7}$$

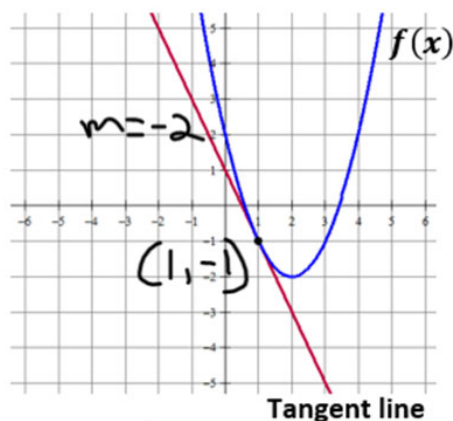
$$y - 12 = -\frac{20}{7}(x + 3)$$

or

$$y + 8 = -\frac{20}{7}(x - 4)$$

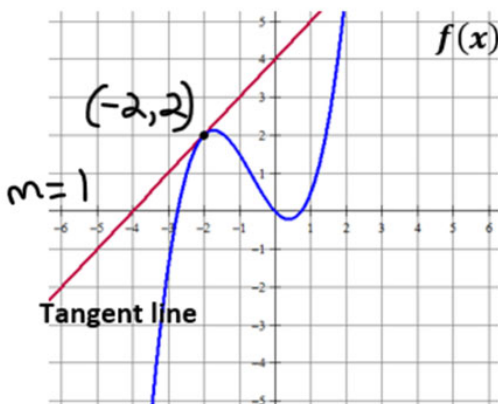
Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



$$y + 1 = -2(x - 1)$$

9. The line tangent to $f(x)$ at $x = -2$



$$y - 2 = 1(x + 2)$$

MULTIPLE CHOICE! Remember slope = $\frac{y_2 - y_1}{x_2 - x_1}$

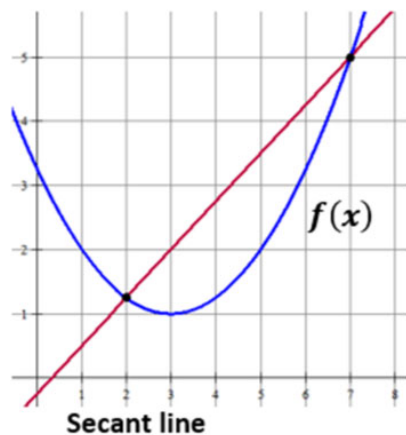
10. Which choice represents the slope of the secant line shown?

A) $\frac{7-2}{f(7)-f(2)}$

B) $\frac{f(7)-2}{7-f(2)}$

C) $\frac{7-f(2)}{f(7)-2}$

D) $\frac{f(7)-f(2)}{7-2}$



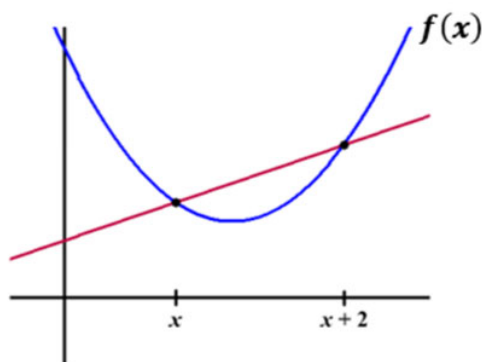
11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$

B) $\frac{f(x+2)-f(x)}{x+2-x}$

C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$



Secant
line

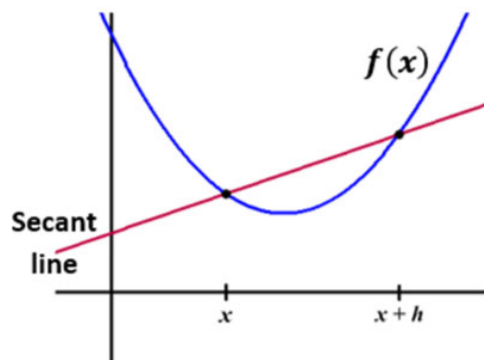
12. Which choice represents the slope of the secant line shown?

A) $\frac{f(x+h)-f(x)}{x-(x+h)}$

B) $\frac{x-(x+h)}{f(x+h)-f(x)}$

C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about the function $f(x)$ is true?

I. $f(2) = 0$

II. $(x+4)$ is a factor of $f(x)$

III. $f(5) = f(-1)$

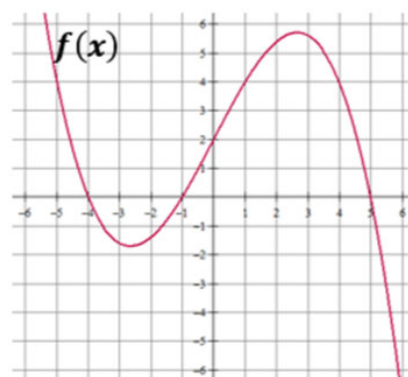
(A) I only

(B) II only

(C) III only

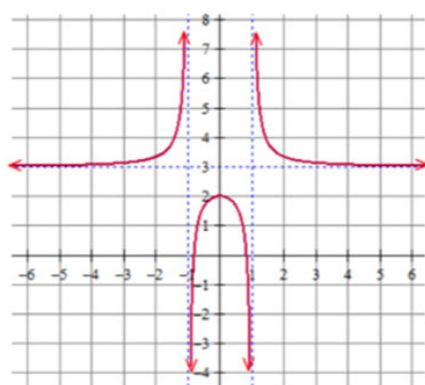
(D) I and III only

(E) II and III only



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Range:

$(-\infty, 2] \cup (3, \infty)$

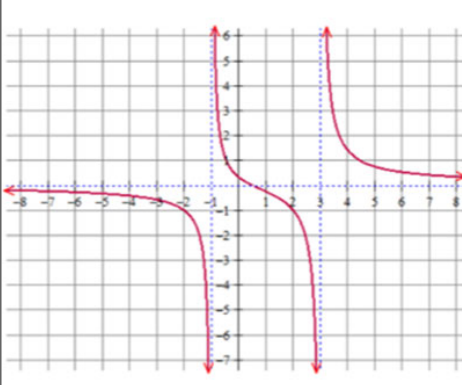
Horizontal Asymptote(s):

$y = 3$

Vertical Asymptotes(s):

$x = 1$
 $x = -1$

15.



Domain: $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

Range:

$(-\infty, \infty)$

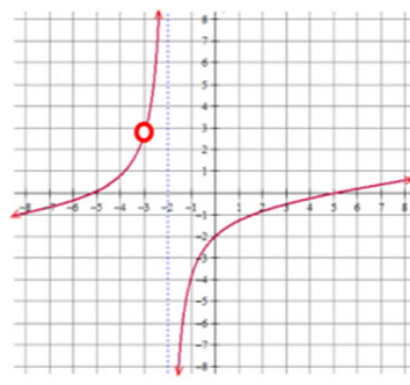
Horizontal Asymptote(s):

$y = 0$

Vertical Asymptotes(s):

$x = -1$
 $x = 3$

16.



Domain: $(-\infty, -2) \cup (-2, \infty)$

Range:

$(-\infty, \infty)$

Horizontal Asymptote(s):

none

Vertical Asymptotes(s):

$x = -2$

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

18. Consider the function: $f(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

- I. $f(x)$ has a vertical asymptote of $x = 2$
- II. $f(x)$ has a vertical asymptote of $x = -2$
- III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$
 $x^{\frac{3}{5}} + (2x)^{\frac{1}{5}}$

20. $\sqrt{x+1}$
 $(x+1)^{\frac{1}{2}}$

21. $\frac{1}{\sqrt{x+1}}$
 $(x+1)^{-\frac{1}{2}}$

22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$
 $x^{-\frac{1}{2}} - 2x^{-1}$

23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$
 $\frac{1}{4}x^{-3} + \frac{1}{2}x^{\frac{3}{4}}$

24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$
 $\frac{1}{4}x^{-\frac{1}{2}} - 2(x+1)^{\frac{1}{2}}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$
 $\frac{1}{\sqrt{x}} - \sqrt{x^3}$

26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$
 $\frac{1}{2\sqrt{x}} + \frac{1}{x}$

27. $3x^{-\frac{1}{2}}$
 $\frac{3}{\sqrt{x}}$

28. $(x+4)^{-\frac{1}{2}}$
 $\frac{1}{\sqrt{x+4}}$

29. $x^{-2} + x^{\frac{1}{2}}$
 $\frac{1}{x^2} + \sqrt{x}$

30. $2x^{-2} + \frac{3}{2}x^{-1}$
 $\frac{2}{x^2} + \frac{3}{2x}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin \frac{\pi}{6}$ $\frac{1}{2}$	32. $\cos \frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$	33. $\sin 2\pi$ 0
34. $\tan \pi$ 0	35. $\sec \frac{\pi}{2}$ undefined	36. $\cos \frac{\pi}{6}$ $\frac{\sqrt{3}}{2}$
37. $\sin \frac{\pi}{3}$ $\frac{\sqrt{3}}{2}$	38. $\sin \frac{3\pi}{2}$ -1	39. $\tan \frac{\pi}{4}$ 1
40. $\csc \frac{\pi}{2}$ 1	41. $\sin \pi$ 0	42. $\cos \frac{\pi}{3}$ $\frac{1}{2}$
43. Find x where $0 \leq x \leq 2\pi$, $\sin x = \frac{1}{2}$ $\frac{\pi}{6}$ and $\frac{5\pi}{6}$	44. Find x where $0 \leq x \leq 2\pi$, $\tan x = 0$ 0, π , and 2π	45. Find x where $0 \leq x \leq 2\pi$, $\cos x = -1$ π

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$ $e^x = 1$ $\ln(e^x) = \ln(1)$ $x = 0$	47. $3e^x + 5 = 8$ $3e^x = 3$ $e^x = 1$ $\ln e^x = \ln 1$ $x = 0$	48. $e^{2x} = 1$ $\ln e^{2x} = \ln(1)$ $2x = 0$ $x = 0$
49. $\ln x = 0$ e^e $x = 1$	50. $3 - \ln x = 3$ $-\ln x = 0$ $\ln x = 0$ e^e $x = 1$	51. $\ln(3x) = 0$ e^e $3x = 1$ $x = \frac{1}{3}$
52. $x^2 - 3x = 0$ $x(x-3) = 0$ $x = 0$ $x = 3$	53. $e^x + xe^x = 0$ $e^x(1+x) = 0$ $e^x = 0$ $1+x = 0$ not possible $x = -1$	54. $e^{2x} - e^x = 0$ $e^x(e^x - 1) = 0$ $e^x = 0$ $e^x - 1 = 0$ not possible $e^x = 1$ $x = 0$

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

56. $\cos x = -1$

$x = \pi$

57. $\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$

58. $2\sin x = -1$
 $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$

59. $\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}$ and $\frac{7\pi}{4}$

60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{2} = \frac{\pi}{6}$ and $\frac{x}{2} = \frac{11\pi}{6}$

$x = \frac{\pi}{3}$

$x = \frac{11\pi}{3}$

Not in the domain interval

61. $\tan x = 0$

$\frac{\sin x}{\cos x} = 0 \rightarrow \sin x = 0$

$x = 0, \pi, 2\pi$

62. $\sin(2x) = 1$

$2x = \frac{\pi}{2}$ and $2x = \frac{5\pi}{2}$

$x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$

63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{4} = \frac{\pi}{3}$ and $\frac{x}{4} = \frac{2\pi}{3}$

$x = \frac{4\pi}{3}$ and $x = \frac{8\pi}{3}$

For each function, determine its domain and range.

Function

Domain

Range

64. $y = \sqrt{x-4}$

$x \geq 4$

$y \geq 0$

65. $y = (x-3)^2$

$\mathbb{R}$ all real numbers

$y \geq 0$

66. $y = \ln x$

$x > 0$

$\mathbb{R}$

67. $y = e^x$

$\mathbb{R}$

$y > 0$

68. $y = \sqrt{4-x^2}$

$-2 \leq x \leq 2$

$0 \leq y \leq 2$

Simplify.

69. $\frac{\sqrt{x}}{x}$

$x^{\frac{1}{2}-1}$
 $x^{-\frac{1}{2}}$

$\frac{1}{\sqrt{x}}$

70. $e^{\ln x}$

x

71. $e^{1+\ln x}$

$e^1 \cdot e^{\ln x}$

ex

72. $\ln 1$

$$0$$

73. $\ln e^7$

$$7$$

74. $\log_3 \frac{1}{3}$
 $\log_3 3^{-1}$

$$-1$$

75. $\log_{1/2} 8$
 $\log_{1/2} (\frac{1}{2})^{-3}$

$$-3$$

76. $\ln \frac{1}{2}$ Calculator needed

$$\approx -0.693$$

77. $27^{2/3}$
 $\sqrt[3]{27^2}$

$$9$$

78. $(5a^{2/3})(4a^{3/2})$

$$20a^{2/3+3/2}$$

$$20a^{13/6}$$

79. $\frac{4xy^{-2}}{12x^{1/3}y^{-5}}$
 $\frac{1}{3}x^{1-1/3}y^{-2-(-5)}$

$$\frac{1}{3}x^{2/3}y^3$$

80. $(4a^{5/3})^{3/2}$
 $\sqrt{4^3} a^{5/3 \cdot 3/2}$

$$8a^{5/2}$$

If $f(x) = \{(3, 5), (2, 4), (1, 7)\}$ $g(x) = \sqrt{x-3}$, then determine each of the following.
 $h(x) = \{(3, 2), (4, 3), (1, 6)\}$ $k(x) = x^2 + 5$

81. $(f+h)(1)$

$$f(1) + h(1)$$

 $7 + 6$

$$13$$

82. $(k-g)(5)$

$$k(5) - g(5)$$

 $(25+5) - (\sqrt{2})$

$$30 - \sqrt{2}$$

83. $f(h(3))$

$$f(2)$$

$$4$$

84. $g(k(7))$

$$g(7^2+5)$$

$$g(54)$$

$$\sqrt{54-3} = \sqrt{51}$$

85. $h(3)$

$$2$$

86. $g(g(9))$

$$g(\sqrt{6})$$

$$\sqrt{\sqrt{6}-3}$$

87. $f^{-1}(4)$

$$2$$

88. $k^{-1}(x)$

$$x = y^2 + 5$$

 $x - 5 = y^2$

$$y = \sqrt{x-5}$$

89. $k(g(x)) = (\sqrt{x-3})^2 + 5$

$$x-3+5$$

$$x+2$$

90. $g(f(2))$

$$g(4) = \sqrt{4-3}$$

$$1$$